Integration- Questions

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

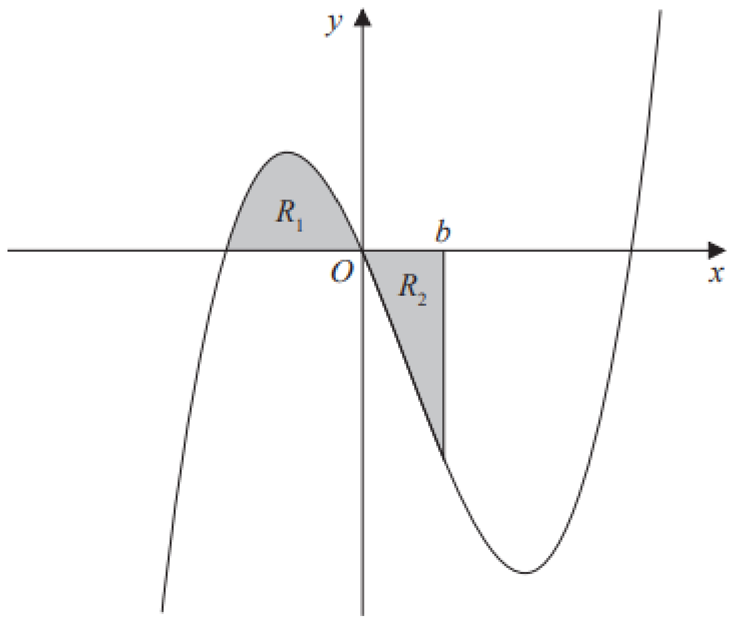

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\frac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places. The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

2.

13. The curve C with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)} \qquad x \in \mathbb{R}, x \neq -3, x \neq 2$$

where p and q are constants, passes through the point $\left(3, \frac{1}{2}\right)$ and has two vertical asymptotes with equations $x = 2$ and $x = -3$

(a) (i) Explain why you can deduce that $q = 4$

(ii) Show that $p = 15$

(3)

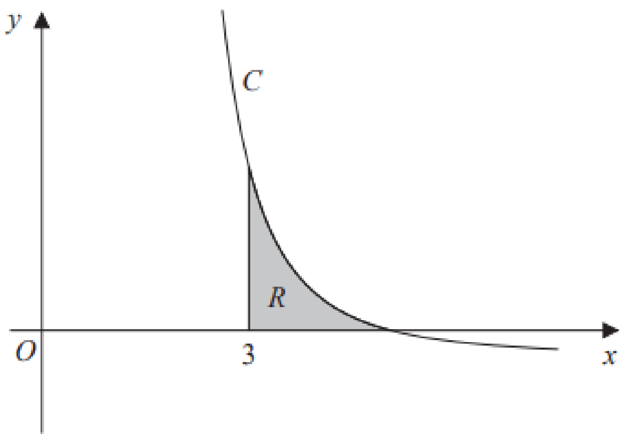

Figure 4

Figure 4 shows a sketch of part of the curve C. The region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the line with equation $x = 3$

(b) Show that the exact value of the area of R is $a\ln 2 + b\ln 3$, where a and b are rational constants to be found.

(8)

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3.

Show that

$$\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x = \frac{32}{15}\left(2+\sqrt{2}\right)$$

(7)

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4.

(a) Given that k is a constant, find

$$\int \left(\frac{4}{x^3} + kx\right)\mathrm{d}x$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2} \left(\frac{4}{x^3} + kx\right)\mathrm{d}x = 8$$

(3)

5.

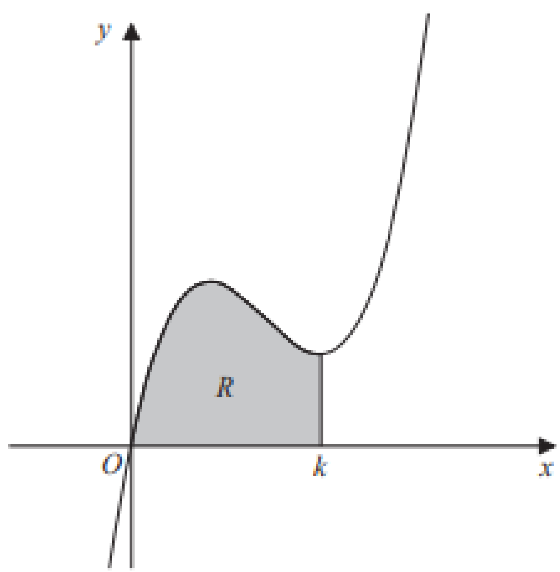

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\dfrac{256}{3}$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

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6.

Find

$$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x$$

giving your answer in its simplest form.

(4)

7.

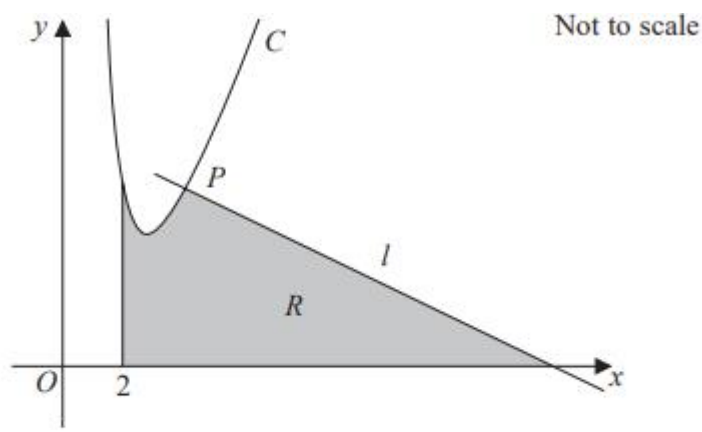

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{32}{x^2} + 3x - 8, \qquad x > 0$$

The point P (4, 6) lies on C.
The line l is the normal to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the line l, the curve C, the line with equation $x = 2$ and the x-axis.

Show that the area of R is 46
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

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8.

Find

$$\int\left(2x^{5}-\frac{1}{4x^{3}}-5\right)\mathrm{d}x$$

giving each term in its simplest form.

(4)

9.

The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, where

$$\mathrm{f}'(x) = 30 + \frac{6-5x^{2}}{\sqrt{x}}$$

Given that the point P (4, −8) lies on C,

(*a*) find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

(*b*) Find $\mathrm{f}(x)$, giving each term in its simplest form.

(5)

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10.

Find

$$\int\left(2x^{4}-\frac{4}{\sqrt{x}}+3\right)\mathrm{d}x$$

giving each term in its simplest form.

(4)

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11.

Given that $y = 4x^3 - \frac{5}{x^2}$, $x \neq 0$, find in their simplest form

(*a*) $\frac{dy}{dx}$,

(3)

(*b*) $\int y \, dx$.

(3)

12.

A curve with equation $y = f(x)$ passes through the point $(4, 9)$.

Given that

$$f'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2, \quad x > 0,$$

(*a*) find $f(x)$, giving each term in its simplest form.

(5)

Point P lies on the curve.

The normal to the curve at P is parallel to the line $2y + x = 0$.

(*b*) Find the x-coordinate of P.

(5)

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13.

Find $\int (8x^3 + 4) \, dx$, giving each term in its simplest form.

(3)

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14.

Find

$$\int \left(10x^4 - 4x - \frac{3}{\sqrt{x}}\right) dx,$$

giving each term in its simplest form.

(4)

15.

$$f'(x) = \frac{(3 - x^2)^2}{x^2}, \quad x \neq 0.$$

(*a*) Show that $f'(x) = 9x^{-2} + A + Bx^2$, where A and B are constants to be found.

(3)

(*b*) Find $f''(x)$.

(2)

Given that the point $(-3, 10)$ lies on the curve with equation $y = f(x)$,

(*c*) find $f(x)$.

(5)

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16.

8.

$$\frac{dy}{dx} = -x^3 + \frac{4x - 5}{2x^3}, \quad x \neq 0.$$

Given that $y = 7$ at $x = 1$, find y in terms of x, giving each term in its simplest form.

(6)

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17.

1. Find

$$\int \left(6x^2 + \frac{2}{x^2} + 5\right) dx,$$

giving each term in its simplest form.

(4)

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18.

1. Given that $y = x^4 + 6x^{\frac{1}{2}}$, find in their simplest form

(*a*) $\dfrac{dy}{dx}$,

(3)

(*b*) $\displaystyle\int y \; dx$.

(3)

19.

7. A curve with equation $y = f(x)$ passes through the point (2, 10). Given that

$$f'(x) = 3x^2 - 3x + 5,$$

find the value of f(1).

(5)

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20.

2. Given that $y = 2x^5 + 7 + \dfrac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

(*a*) $\dfrac{dy}{dx}$,

(3)

(*b*) $\displaystyle\int y \; dx$.

(4)

21.

6. Given that $\dfrac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 3x^q$,

(*a*) write down the value of p and the value of q.

(2)

Given that $\dfrac{dy}{dx} = \dfrac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$ and that $y = 90$ when $x = 4$,

(*b*) find y in terms of x, simplifying the coefficient of each term.

(5)

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22.

2. Find

$$\int (12x^5 - 3x^2 + 4x^{\frac{1}{3}}) \, dx,$$

giving each term in its simplest form.

(5)

23.

7. The curve with equation $y = f(x)$ passes through the point $(-1, 0)$.

Given that

$$f'(x) = 12x^2 - 8x + 1,$$

find $f(x)$.

(5)

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24.

2. Find

$$\int (8x^3 + 6x^{\frac{1}{2}} - 5) \, dx,$$

giving each term in its simplest form.

(4)

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25.

4. $$\frac{dy}{dx} = 5x^{-\frac{1}{2}} + x\sqrt{x}, \qquad x > 0.$$

Given that $y = 35$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

(7)

26.

6.

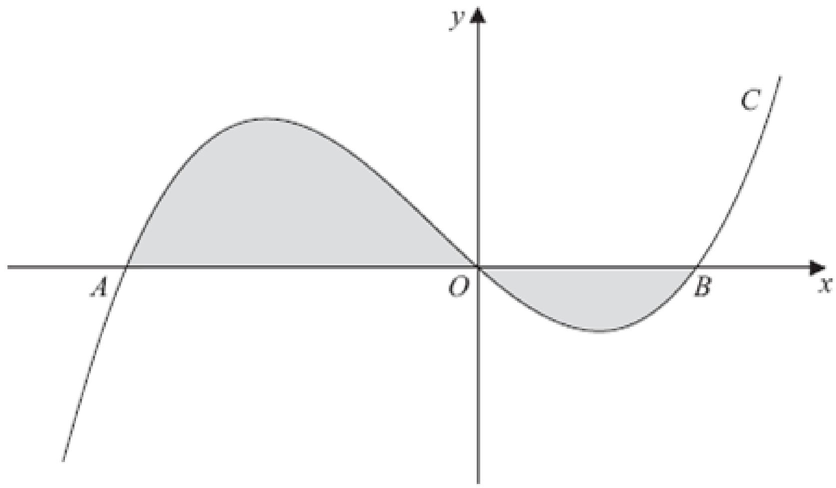

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2).$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

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27.

5.

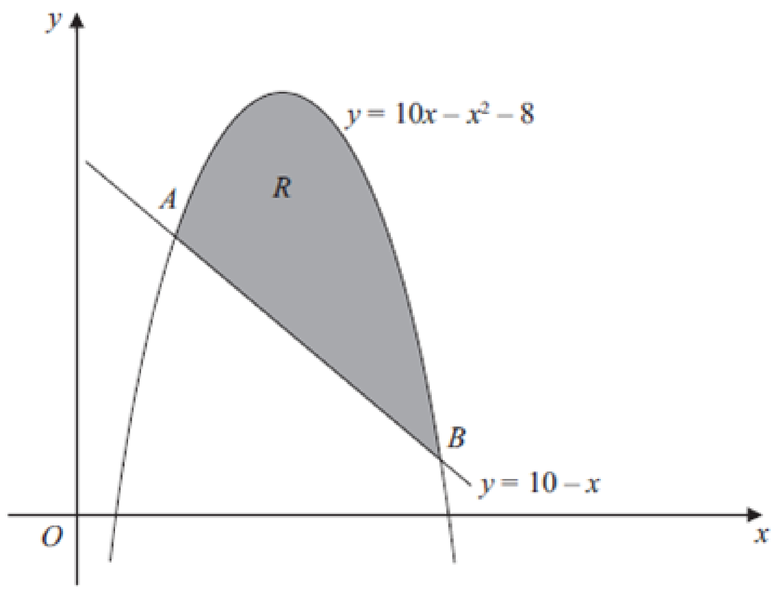

Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$.

The line and the curve intersect at the points A and B, and O is the origin.

(*a*) Calculate the coordinates of A and the coordinates of B.

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(*b*) Calculate the exact area of R.

(7)

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28.

9.

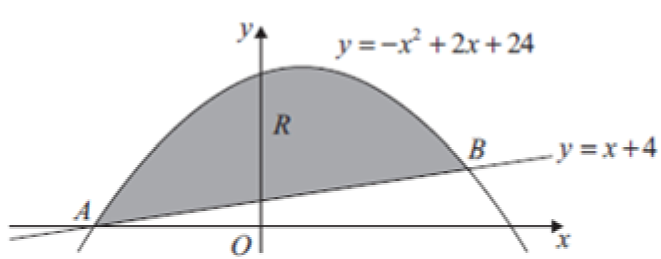

Figure 3

The straight line with equation $y = x + 4$ cuts the curve with equation $y = -x^2 + 2x + 24$ at the points A and B, as shown in Figure 3.

(a) Use algebra to find the coordinates of the points A and B.

(4)

The finite region R is bounded by the straight line and the curve and is shown shaded in Figure 3.

(b) Use calculus to find the exact area of R.

(7)

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29.

4.

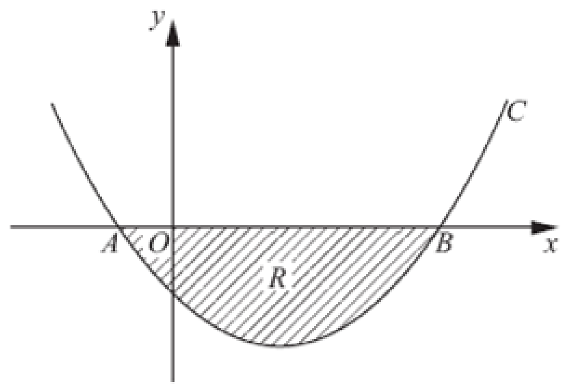

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x + 1)(x - 5).$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

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30.

8.

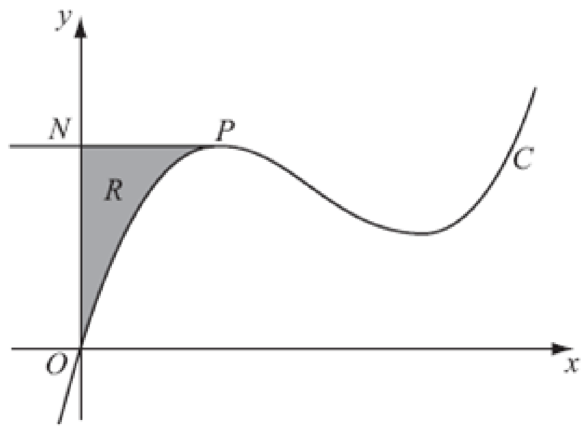

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(*a*) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(*b*) Use calculus to find the exact area of R.

(6)

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31.

7.

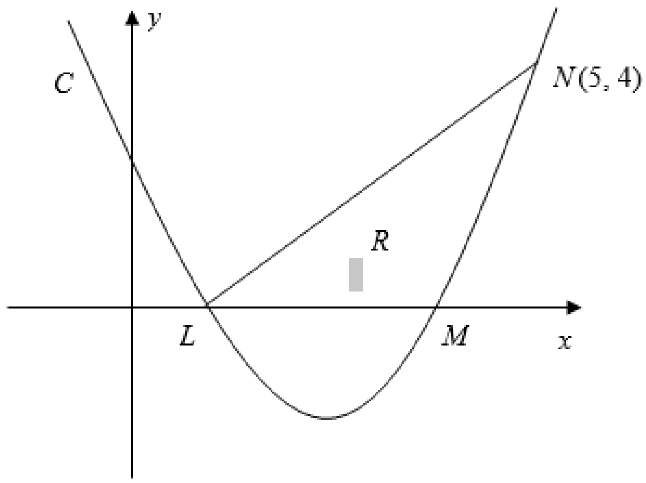

Figure 2

The curve C has equation $y = x^2 - 5x + 4$. It cuts the x-axis at the points L and M as shown in Figure 2.

(*a*) Find the coordinates of the point L and the point M.

(2)

(*b*) Show that the point $N(5, 4)$ lies on C.

(1)

(*c*) Find $\int (x^2 - 5x + 4) \ \mathrm{d}x$.

(2)

The finite region R is bounded by LN, LM and the curve C as shown in Figure 2.

(*d*) Use your answer to part (*c*) to find the exact value of the area of R.

(5)